ΘΕΜΑ 1ο

Α. θεωρία σχολικό βιβλίο σελ. 28

Β. iii

Γ. α)Λ   β)Σ   γ)Λ   δ)Σ   ε)Λ

Δ. α. $\sigma\upsilon\nu(\alpha - \beta) = \sigma\upsilon\nu\alpha \cdot \sigma\upsilon\nu\beta + \eta\mu\alpha \cdot \eta\mu\beta$

β. $\ell oge \cdot \ell n10 = \dfrac{\ell ne}{\ell n10} \cdot \ell n10 = 1$

γ. $\ell og\dfrac{\theta_1}{\theta_2} = \ell og\theta_1 - \ell og\theta_2$

ΘΕΜΑ 2ο

Α. Πρέπει : $x^2 = 1 \cdot (2 - x) \Leftrightarrow x^2 = 2 - x \Leftrightarrow x^2 + x - 2 = 0$

$$\Delta = 1 + 8 = 9$$

$$x = \frac{-1 \pm 3}{2} = \begin{cases} 1 \\ -2 \end{cases}$$

Β. Πρέπει : $\left.\begin{array}{l} P(1) = 0 \\ P(-2) = 0 \end{array}\right\} \Leftrightarrow \left.\begin{array}{l} 1^4 + (\alpha - \beta) \cdot 1^3 - (2\alpha - 3\beta) \cdot 1^2 + 1 - 2 = 0 \\ (-2)^4 + (\alpha - \beta)(-2)^3 - (2\alpha - 3\beta)(-2)^2 + (-2) - 2 = 0 \end{array}\right\}$

$\Leftrightarrow \left.\begin{array}{l} 1 + \alpha - \beta - 2\alpha + 3\beta + 1 - 2 = 0 \\ 16 - 8\alpha + 8\beta - 8\alpha + 12\beta - 2 - 2 = 0 \end{array}\right\} \Leftrightarrow \left.\begin{array}{l} -\alpha + 2\beta = 0 \\ -16\alpha + 20\beta = -12 \end{array}\right\} \Leftrightarrow$

$\left.\begin{array}{l} -\alpha + 2\beta = 0 \\ -4\alpha + 5\beta = -3 \end{array}\right\} \Leftrightarrow \left.\begin{array}{l} \alpha = 2\beta \\ -8\beta + 5\beta = -3 \end{array}\right\} \Leftrightarrow \left.\begin{array}{l} \alpha = 2 \\ \beta = 1 \end{array}\right.$

ΘΕΜΑ 3ο

Α. Πρέπει : $\alpha_2 - \alpha_1 = \alpha_3 - \alpha_2 \Leftrightarrow \dfrac{1}{2}\sigma\upsilon\nu 2\alpha - 1 = -2\eta\mu^2\alpha - \dfrac{1}{2}\sigma\upsilon\nu 2\alpha \Leftrightarrow \sigma\upsilon\nu 2\alpha = 1 - 2\eta\mu^2\alpha$ (ισχύει)

Β. $\omega = \alpha_2 - \alpha_1 = \dfrac{1}{2}\sigma\upsilon\nu 2\alpha - 1 = \dfrac{1 - 2\eta\mu^2\alpha}{2} - 1 = -\eta\mu^2\alpha - \dfrac{1}{2}$

$S4 = -2 \Leftrightarrow \dfrac{4}{2}\left[2\cdot 1 + (4-1)\cdot\left(-\eta\mu^2\alpha - \dfrac{1}{2}\right)\right] = -2 \Leftrightarrow 4 - 6\eta\mu^2\alpha - 3 = -2$

$6\eta\mu^2\alpha = 3 \Leftrightarrow \eta\mu^2\alpha = \dfrac{1}{2}$

$\eta\mu\alpha = \dfrac{\sqrt{2}}{2} \qquad \eta\mu\alpha = \eta\mu\dfrac{\pi}{4}$

$\alpha = 2\kappa\pi + \dfrac{\pi}{4}$ η $\alpha = 2\kappa\pi + \dfrac{3\pi}{4}$ ή

$\eta\mu\alpha = -\dfrac{\sqrt{2}}{2}$

$\eta\mu\alpha = \eta\mu\left(-\dfrac{\pi}{4}\right)$

$\alpha = 2\kappa\pi - \dfrac{\pi}{4}$ η $\alpha = 2\kappa\pi + \dfrac{5\pi}{4}$, $\kappa \in z$

Γ. $S_{103} = \dfrac{103}{2}\left[2\cdot 1 + (103-1)\cdot\left(-\eta\mu^2\dfrac{\pi}{4} - \dfrac{1}{2}\right)\right] = 103[1 + 51\cdot(-1)] = 103\cdot(-50) = -5150$

Δ. $S_5 = \dfrac{5}{2}\left[2\cdot 1 + (5-1)\cdot\left(-\eta\mu^2\alpha - \dfrac{1}{2}\right)\right] = 5[1 - 2\eta\mu^2\alpha - 1] = -10\eta\mu^2\alpha \neq 0$, $\alpha \in \left(0, \dfrac{\pi}{2}\right)$

(Αν $S_5 = 0 \Leftrightarrow \cdots \eta\mu^2\alpha = 0$ Άτοπο)

Άρα ο βαθμός του πολυωνύμου είναι 5.

ΘΕΜΑ 4ο

Α. Πρέπει $2e^{2x+1} + e^{x+1} > 0$ Άρα $x \in R$ δηλαδή $A_f = R$

$f(0) = \ell n(2e^{2\cdot 0+1} + e^{0+1}) = \ell n(2e + e) = \ell n 3e = \ell n 3 + \ell n e = \ell n 3 + 1$

Β. $f(x) = 1 \Leftrightarrow \ell n(2e^{2\cdot x+1} + e^{x+1}) = \ell n e \xleftarrow{\ell nx:1-1} 2e^{2\cdot x+1} + e^{x+1} = e$

$2e\cdot e^{2x} + e\cdot e^x - e = 0 \Leftrightarrow e(2e^{2x} + e^x - 1) = 0 \Leftrightarrow \begin{matrix} 2(e^x)^2 + e^x - 1 = 0 \\ \theta\varepsilon\tau\omega \quad e^x = y \end{matrix} \Rightarrow$

$2y^2 + y - 1 = 0 \cdots$ $y = -1$ η $y = \dfrac{1}{2}$

Άρα $e^x = -1$ ΑΔΥΝΑΤΗ ή $e^x = \dfrac{1}{2} \Leftrightarrow \ell n e^x = \ell n \dfrac{1}{2} \Leftrightarrow x\cdot \ell n e = \ell n 1 - \ell n 2 \Leftrightarrow x = -\ell n 2$

Γ. $f(x) < 1 \Leftrightarrow \cdots 2e^{2x} + e^x - 1 < 0$, $-1 < e^x < \dfrac{1}{2}$, $e^x > -1 \Leftrightarrow x \in R$

$e^x < \dfrac{1}{2} \xleftarrow{\ell nx\uparrow} \ell n e^x < \ell n \dfrac{1}{2} \Leftrightarrow x < -\ell n 2$

Άρα $x \in (-\infty, -\ell n 2)$