



Α' ΛΥΚΕΙΟΥ

ΑΛΓΕΒΡΑ

ΑΠΑΝΤΗΣΕΙΣ

ΘΕΜΑ 1ο

- A.** Ορισμός σχολικού βιβλίου, σελ. 63.
B. Απόδειξη, σελ. 45, σχολικού βιβλίου.
Γ.
 - α.** Λ
 - β.** Σ
 - γ.** Λ
 - δ.** Σ
 - ε.** Λ

ΘΕΜΑ 2ο

α.

$$D = \begin{vmatrix} \lambda + 2 & 5 \\ 1 & \lambda - 2 \end{vmatrix} = \lambda^2 - 4 - 5 = \lambda^2 - 9 = (\lambda + 3)(\lambda - 3)$$

$$Dx = \begin{vmatrix} 5 & 5 \\ -5 & \lambda - 2 \end{vmatrix} = 5(\lambda - 2) + 25 = 5\lambda - 10 + 25 = 5\lambda + 15 = 5(\lambda + 3)$$

$$Dy = \begin{vmatrix} \lambda + 2 & 5 \\ 1 & -5 \end{vmatrix} = -5(\lambda + 2) - 5 = -5\lambda - 10 - 5 = -5\lambda - 15 = -5(\lambda + 3)$$

- β.** Αν $D \neq 0$, τότε $(\lambda + 3)(\lambda - 3) \neq 0$, οπότε $\lambda \neq -3$ και $\lambda \neq 3$, επομένως το σύστημα έχει μοναδική λύση την:

$$x = \frac{D_x}{D} = \frac{5(\lambda + 3)}{(\lambda + 3)(\lambda - 3)} = \frac{5}{\lambda - 3}$$

$$y = \frac{D_y}{D} = \frac{-5(\lambda + 3)}{(\lambda + 3)(\lambda - 3)} = \frac{-5}{\lambda - 3}$$

$$\text{Αν } D = 0 \Leftrightarrow \lambda = -3 \text{ ή } \lambda = 3$$

$$\text{i)} \quad \left. \begin{array}{l} \lambda = -3 \Rightarrow -x + 5y = 5 \\ x - 5y = -5 \end{array} \right\} \cdot (-1) \quad \left. \begin{array}{l} x - 5y = -5 \\ x - 5y = -5 \end{array} \right\} \Leftrightarrow x - 5y = -5 \Leftrightarrow$$

$$x = 5y - 5, \quad y \in \mathbb{R}$$

$$(x, y) = (5y - 5, y), \quad y \in \mathbb{R}, \quad \text{άπειρες λύσεις}$$

$$\text{ii)} \quad \left. \begin{array}{l} \lambda = 3 \Rightarrow 5x + 5y = 5 \\ x + y = -5 \end{array} \right\} : 5 \quad \left. \begin{array}{l} x + y = 1 \\ x + y = -5 \end{array} \right\} \Leftrightarrow 1 = -5$$

αδύνατο

γ.

$$x_0 = \frac{5}{\lambda - 3}, \quad y_0 = \frac{-5}{\lambda - 3}$$

$$\left| \frac{5}{x_0} - \frac{5}{y_0} \right| = 1 \Leftrightarrow \left| \frac{5}{\frac{5}{\lambda-3}} - \frac{5}{-\frac{5}{\lambda-3}} \right| = 1 \Leftrightarrow |\lambda - 3 + \lambda - 3| = 1 \Leftrightarrow$$

$$|2\lambda - 6| = 1 \Leftrightarrow 2\lambda - 6 = 1 \quad \text{ή} \quad 2\lambda - 6 = -1 \Leftrightarrow \lambda = \frac{7}{2} \quad \text{ή} \quad \lambda = \frac{5}{2} \quad (\text{δεκτές})$$

ΘΕΜΑ 3ο

α. Αφού η εξίσωση έχει δύο ρίζες άνισες, τότε

$$\Delta > 0 \Leftrightarrow (1 - \lambda)^2 - 4 > 0 \Leftrightarrow (1 - \lambda)^2 > 4 \Leftrightarrow |1 - \lambda| > 2$$

$$\text{β.} \quad |1 - \lambda| > 2 \Leftrightarrow 1 - \lambda > 2 \quad \text{ή} \quad 1 - \lambda < -2 \Leftrightarrow 1 - 2 > \lambda \quad \text{ή} \quad 1 - 2 < \lambda$$

$$\lambda < -1 \quad \text{ή} \quad \lambda > 3$$

γ. Από τους τύπους του Vieta

$$K = x_1 + x_2 = -\frac{\beta}{\alpha} = -\frac{1 - \lambda}{1} = \lambda - 1$$

$$\Lambda = x_1 \cdot x_2 = \frac{\gamma}{\alpha} = 1$$

$$M = \frac{1}{x_1} + \frac{1}{x_2} = \frac{x_1 + x_2}{x_1 \cdot x_2} = \frac{\lambda - 1}{1} = \lambda - 1$$

δ. $\lambda x_1 x_2 (x_1 + x_2) + 3(x_1 + x_2) = 5$

$$\lambda \cdot 1 \cdot (\lambda - 1) + 3 \cdot (\lambda - 1) = 5$$

$$\lambda^2 - \lambda + 3\lambda - 3 - 5 = 0$$

$$\lambda^2 + 2\lambda - 8 = 0$$

$$\left. \begin{array}{l} \lambda = 2 \quad \text{ή} \quad \lambda = -4 \\ \lambda < -1 \quad \text{ή} \quad \lambda > 3 \end{array} \right\} \Leftrightarrow \lambda = -4, \text{ δεκτή}$$

ΘΕΜΑ 4ο

α.

$$\left. \begin{array}{l} f(-2) = f(4) \\ f(2) = f(-1) \end{array} \right\} \Leftrightarrow \left. \begin{array}{l} 2\alpha \cdot (-2) - 5 = 4 + \beta \\ 2 + \beta = 2\alpha(-1) - 5 \end{array} \right\} \Leftrightarrow \left. \begin{array}{l} -4\alpha - 5 = 4 + \beta \\ 2 + \beta = -2\alpha - 5 \end{array} \right\} \Leftrightarrow$$

$$-4\alpha - \beta = 9$$

$$2\alpha + \beta = -7$$

$$\left. \begin{array}{l} -2\alpha = 2 \Leftrightarrow \alpha = -1 \\ 2\alpha + \beta = -7 \end{array} \right\} \Leftrightarrow 2 \cdot (-1) + \beta = -7 \Leftrightarrow \beta = -5$$

β. $\lambda \varepsilon_1 = \lambda \varepsilon_2 \Leftrightarrow \lambda^4 + 2 = 13\lambda^2 - 34 \Leftrightarrow \lambda^4 - 13\lambda^2 + 36 = 0 \Leftrightarrow$

$$\left. \begin{array}{l} (\lambda^2)^2 - 13\lambda^2 + 36 = 0 \\ \text{θέτω } \lambda^2 = \omega \end{array} \right\} \Leftrightarrow \omega^2 - 13\omega + 36 = 0 \Leftrightarrow \omega_1 = 4 \quad \text{ή} \quad \omega_2 = 9$$

$$\lambda^2 = 4 \Leftrightarrow \lambda = \pm 2$$

$$\lambda^2 = 9 \Leftrightarrow \lambda = \pm 3$$

γ. το πεδίο ορισμού της f είναι $A = [-5, 5)$

• $-5 \leq x < 2$

$$f(x) = 1 \Leftrightarrow -2x - 5 = 1 \Leftrightarrow -2x = 6 \Leftrightarrow x = -3 \quad \text{δεκτή}$$

• $2 \leq x < 5$

$$f(x) = 1 \Leftrightarrow x - 5 = 1 \Leftrightarrow x = 6 \quad \text{απορρίπτεται}$$